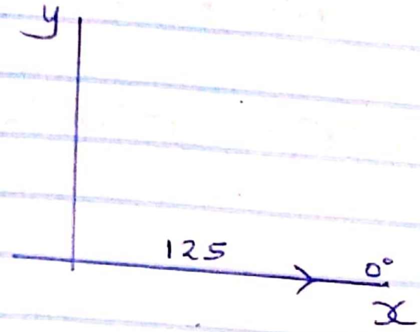
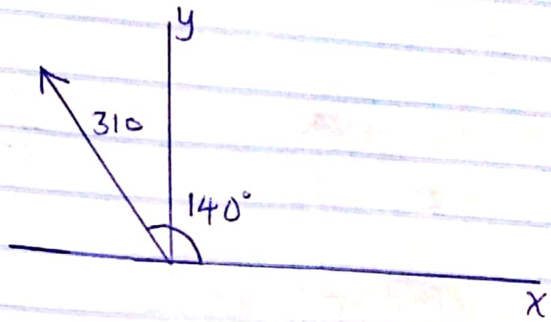
1. $\vec{W} = \vec{u} + \vec{V}$, $\vec{u} = 125@0^\circ$ and $\vec{V} = 310@140^\circ$

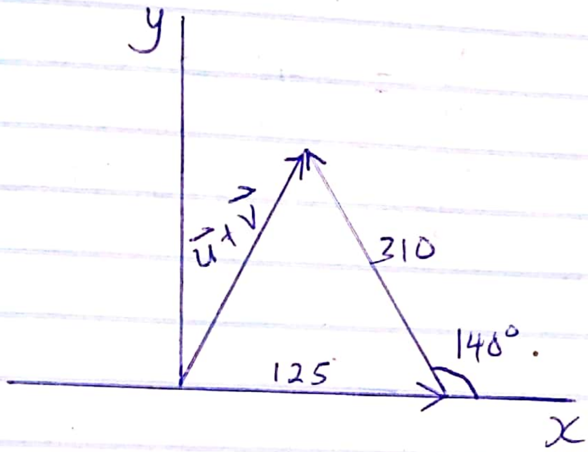
$\vec{u} = 125@0^\circ$

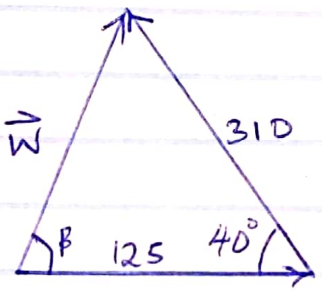
$\vec{V} = 310@140^\circ$





$\vec{u} + \vec{V}$.





Magnitude:

$\vec{W} = c^2 = a^2 + b^2 - 2ab\cos\theta$

$c^2 = 125^2 + 310^2 - 2 \times 125 \times 310 \cos 40^\circ$

$c^2 = 15625 + 96100 - 59368.444$

$c^2 = 52{,}356.556$

$c = 228.816$

Direction: find angle β: $\frac{\sin\beta}{310} = \frac{\sin 40^\circ}{228.816} \Rightarrow \sin\beta = \frac{310 \cdot \sin 40^\circ}{228.816}$

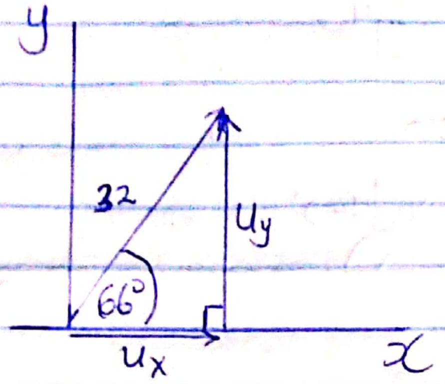
$\Rightarrow \beta = \sin^{-1} 0.8708 = 60.6^\circ$

Angle made by $\vec{W}$ relative to positive x-axis: 60.6°

$\Rightarrow \vec{W} = \cancel{60.6}\ 228.816 @ 60.6^\circ$

2. $\vec{w} = \vec{u} + \vec{v}$, $\vec{u} = 32 @ 66°$ and $\vec{v} = 19 @ -15°$

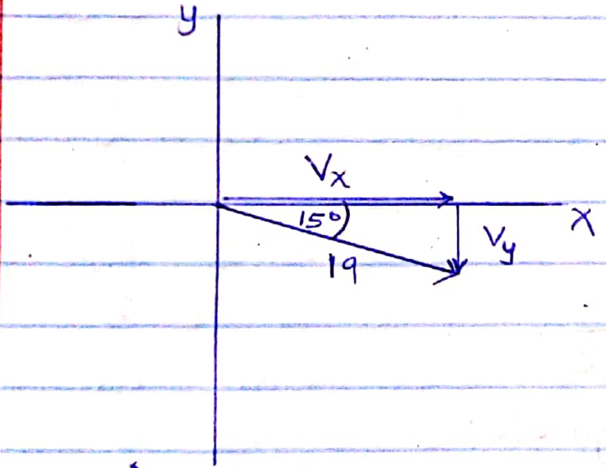
$\vec{u} = 32 @ 66°$



$\frac{U_x}{32} = \cos 66°$ or $U_x = 32 \cdot \cos 66° = 13.02$

$\frac{U_y}{32} = \sin 66°$ or $U_y = 32 \sin 66° = 29.23$

$\vec{v} = 19 @ -15°$



$\frac{V_x}{19} = \cos -15° \Rightarrow V_x = 19 \cdot \cos -15° = \cancel{4.92}$ -18.35

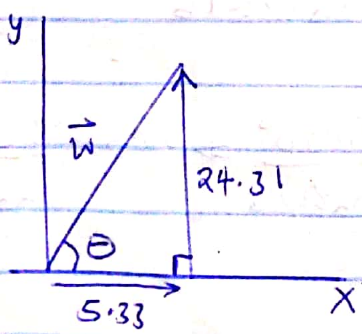
$\frac{V_y}{19} = \sin -15° \Rightarrow V_y = 19 \cdot \sin -15° = -4.92$

If $\vec{w} = \vec{u} + \vec{v}$ then $W_x = U_x + V_x$ and $W_y = U_y + V_y$

$W_x = 13.02 - 18.35 = -5.33$

$W_y = 29.23 - 4.92 = 24.31$

So



$|\vec{w}| = \sqrt{24.31^2 + 5.33^2} = 24.89$

$\tan \theta = \frac{24.89}{5.33} = 4.67$ or $\theta = \tan^{-1} 4.67 = 1.4°$

$\vec{w} = 24.89 @ 1.4°$

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3 $\vec{u} = 6i - 5j$ and $\vec{v} = -8i - 3j$

(a) $3\vec{u} = 3(6\hat{i} - 5\hat{j}) = 18\hat{i} - 15\hat{j}$

(b) $\vec{u} - \vec{v} = (6\hat{i} - 5\hat{j}) - (-8\hat{i} - 3\hat{j})$

$= 6\hat{i} + 8\hat{i} - 5\hat{j} + 3\hat{j}$

$= 14\hat{i} - 2\hat{j}$

(c) $\frac{1}{2}\vec{v} = \frac{1}{2}(-8\hat{i} - 3\hat{j}) = -4\hat{i} - \frac{3}{2}\hat{j}$

4 $\vec{u} = 12i - 5j$

Let the unit vector which is parallel to $\vec{u}$ be $\vec{v} = (a, b)$

$|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}|\sin\theta$

If $\vec{u} \parallel \vec{v}$, then $\theta = 0°$ and $\sin 0° = 0$

So if $\vec{u} \parallel \vec{v}$, then $|\vec{u} \times \vec{v}| = 0$

Take $(12i - 5j) \times (ai + bj) = 0$

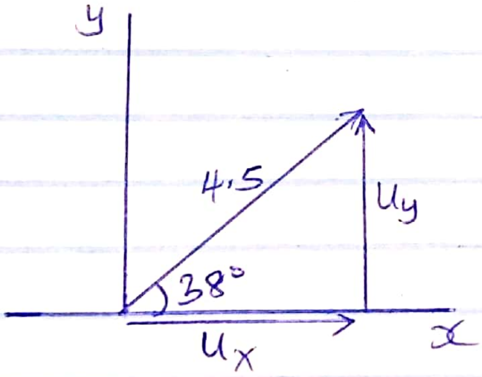
$$\begin{vmatrix} i & j & k \\ 12 & -5 & 0 \\ a & b & 0 \end{vmatrix} = 0 - 0 + 12b + 5a = 0 \Rightarrow 5a = -12b$$

If $a = 1$, then $b = -\frac{5}{12}$ or $\vec{v} = \left(1, -\frac{5}{12}\right)$

So $\vec{v} = \left(1, -\frac{5}{12}\right)$ is one vector that is parallel to $\vec{u} = (12, -5)$

5 $\vec{u} = 4.5@38°$ and $\vec{v} = 7.4@71°$

$\vec{u} = 4.5@38°$

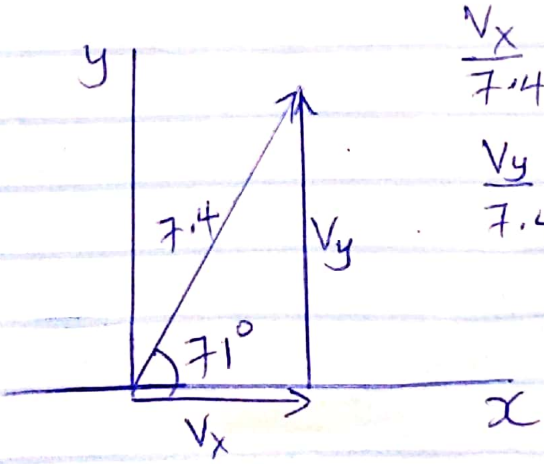


$\frac{u_x}{4.5} = \cos 38° \Rightarrow u_x = 4.5\cos 38° = 3.55$

$\frac{u_y}{4.5} = \sin 38° \Rightarrow u_y = 4.5\sin 38° = 2.77$

Then $\vec{u} = 3.55i + 2.77j$

$\vec{v} = 7.4@71°$



$\frac{v_x}{7.4} = \cos 71° \Rightarrow v_x = 7.4\cos 71° = 2.41$

$\frac{v_y}{7.4} = \sin 71° \Rightarrow v_y = 7.4\sin 71° = 7.0$

Then $\vec{v} = 2.41i + 7j$

Therefore $\vec{u}\cdot\vec{v} = (3.55\hat{i} + 2.77\hat{j})\cdot(2.41\hat{i} + 7\hat{j})$

$\vec{u}\cdot\vec{v} = (3.55 \times 2.41)\hat{i} + (2.77 \times 7)\hat{j}$

$\vec{u}\cdot\vec{v} = 8.56i + 19.39j$

6. $u = (5,7) = 5\hat{i} + 7\hat{j}$ and $v = (2,-9) = 2\hat{i} - 9\hat{j}$

$\vec{u}\cdot\vec{v} = (5\hat{i} + 7\hat{j})\cdot(2\hat{i} - 9\hat{j})$

$= (5\times 2)\hat{i} + (7\times -9)\hat{j}$

$= 10\hat{i} - 63j$

7. $\cos\theta = \dfrac{u\cdot v}{|u||v|}$

$\cos\theta = \dfrac{(5,7)\cdot(2,-9)}{|(5,7)||(2,-9)|}$

$\cos\theta = \dfrac{10-63}{(\sqrt{74})(\sqrt{85})}$

$\cos\theta = \dfrac{-53}{79.31}$

$\theta = \cos^{-1} -0.6683 = 131.9°$

8. $\vec{u} = 12i - 5j = (12, -5)$

Let $\vec{w} = (a, b)$

$\vec{w}\cdot\vec{v} = |\vec{w}||\vec{v}|\cdot\cos\theta$

If $\vec{w} \perp \vec{u}$, then $\theta = 90°$ and $\cos 90° = 0$

so if $\vec{w} \perp \vec{u}$ then $\vec{u}\cdot\vec{w} = 0$

Take $(12, -5)\cdot(a, b) = 0$

$12a - 5b = 0 \Rightarrow 12a = 5b$

If $a = 1$, then $b = 12/5$ or $\vec{w} = (1, 12/5)$

So $\vec{w} = (1, 12/5)$ is one vector that is perpendicular to $\vec{u} = (12, -5)$

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